MA 8109 SUMMARY LEC 7

Int. theory → probab. theory → stoch. proc. → stoch. int. → SDEs → ...

Brownian motion (B.M.)

A stochastic process $B_t : \Omega \times [0,\infty) \to \mathbb{R}^n$ such that

(i) $B_0 = 0$ a.s.

(ii) $\Delta B_k = B_{t_{k+1}} - B_{t_k} \sim N(\vec{0}, (t_{k+1} - t_k)\underset{n\times n}{I})$

(iii) $\Delta B_1, \dots, \Delta B_k$ independent $\forall k$, $t_1 < \dots < t_n$

B_t B.M. $\iff$ B_t Gaussian, $E(B_t) = 0$, $E(B_s \cdot B_t) = \min(s,t)$